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Optimization of Fuzzy Zero- Base Budgeting

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Abstract

Zero-Base Budgeting (ZBB) is a very well-known method for the selection and management of budgets and is widely used by companies and government agencies. In this paper, a new method for modelling ZBB in fuzzy environment is described. Triangular fuzzy numbers are used for describing the imprecise budget data. In addition, an alternative approach is proposed for people who need to be more precise in their requirements. The efficiency of the proposed method is illustrated by numerical example using triangular fuzzy numbers and possibility theory.

Keywords: Zero-base budgeting, Triangular fuzzy numbers, Interval confidence, Fuzzy threshold.

1 | Introduction

An enormous systems of budgeting have introduced into practice over the last sixty years. While more popular budgeting systems as incrementalism and planning programming budgeting systems have continued popularity and resilience as budgeting systems, ZBB has experienced a resurgence in popularity, as governments and public organizations alike seek to control wasteful spending within their departments [5]. Pyhrr [11] identified four basics steps that constitutes the ZBB problems. Shayne [12] confirmed that government across the global facing budget cuts and increased public serenity, government agencies have used alternative budgeting method such as zero- based budgeting instead of the line item and incremental budgeting. Ibrahim et al. [4] performed study about the prediction of the possibility of adopting ZBB system in Borno state, this study considered viability as predictor variable that has perceived to have contributed to adopting of the ZBB in the state. Ibrahim [5] provides users with a step by step guide to designing ZBB for public organizations. This work beginning by describing the foundations of ZBB as well as providing a brief comparative exploration of ZBB alongside other contemporary budgeting systems. Al-attara et al. [1] developed the budget systems in the sub units of government aim to use zero based budget system, make the necessary steps to implement, and to reduce the waste in the public fund.

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In literature, first of all, Zadeh [13] proposed the philosophy of fuzzy sets. Decision making in a fuzzy environment, developed by Bellman and Zadeh [2] has an improvement and a great help in the management decision problems. Khalifa and Alodhaibi [7] introduced a new method for ZBB modelling using pentagonal fuzzy number. Hatami-Marbini et al. [10] extended a fuzzy network data envelopment analysis model to measure maturity levels of a performance based budgeting systems. Imprecise budget data are described using pentagonal fuzzy numbers Zimmermann (1978) introduced fuzzy programming and linear programming with multiple objective functions. Later several researchers worked in fuzzy set theory [6]. The theory and applications of fuzzy sets systems and fuzzy mathematical models have introduced by Dubois [3], Kaufmann and Gupta [8].

In this paper, a new method for modelling ZBB in fuzzy environment is described, where triangular fuzzy numbers characterize the budget data.

The outlay of the paper is organized as follows: In the next section, some preliminaries and notations needed are presented. Section 3 provides a numerical example to illustrate the efficiency of the proposed method. Section 4 introduces an alternative approach. Section 5 introduces discussion about the results. Finally, some concluding remarks are reported in Section 6.

2 | Preliminaries

In order to easily discuss the problem, it recalls basic rules and findings related to fuzzy set, fuzzy numbers, triangular fuzzy numbers, and arithmetic operations.

Definition 1 ([9]). Let $M = X_1 \times X_2 \times \ldots \times X_n$ be the Cartesian product, μ_i be the fuzzy set in X_i, and $f: X \to Y$ be a mapping. Then, the fuzzy set A in Y can be defined using the extension principle as:

$$\widetilde{A}\ y) = \begin{cases} \underset{x_1,\ldots,x_n)\in f^{-1}\ y)}{\mathrm{Sup}} \min\{\mu_1\ x_1), \ldots, \mu_n\ x_n)\}, & f^{-1}\ y) \neq \emptyset, \\ 0, & f^{-1}\ y) = \emptyset. \end{cases}$$

Definition 2 ([9]). Let A and B be two fuzzy sets, the algebraic operations are defined as:

I. Addition: $A\ +)B$

$$\mu_{\widetilde{A}\ +)B}\ z) = \underset{z=x+y}{\mathrm{Sup}} \min\{\mu_{\widetilde{A}}, \mu_B\}, x \in \widetilde{A}, y \in B.$$

II. Subtraction: $A\ -)B$

$$\mu_{\widetilde{A}\ -)B}\ z) = \underset{z=x-y}{\mathrm{Sup}} \min\{\mu_{\widetilde{A}}, \mu_B\}, x \in \widetilde{A}, y \in B.$$

III. Multiplication: $\widetilde{A}\ .)B$

$$\mu_{\widetilde{A}\ .)B}\ z) = \underset{z=x.y}{\mathrm{Sup}} \min\{\mu_{\widetilde{A}}, \mu_B\}, x \in \widetilde{A}, y \in B.$$

IV. Division: $\widetilde{A}\ /)B$

$$\mu_{\widetilde{A}\ /)B}\ z) = \underset{z=x/y}{\mathrm{Sup}} \min\{\mu_{\widetilde{A}}, \mu_B\}, x \in \widetilde{A}, y \in B.$$

Definition 3 ([13]). Fuzzy number: A fuzzy number F is a fuzzy set with a membership function defined as

$\pi_F\ x): \Re \to [0,1]$, and satisfies:

El-Morsy| Com. Alg. Num. Dim. 1(4) (2022) 147-154

I. F is fuzzy convex, i.e., $\pi_F(\delta x + (1-\delta) y) \geq min\{\pi_F(x), \pi_F(y)\}$; $\forall\ x, y \in \Re; 0 \leq \delta \leq 1$.

II. F is normal, i.e., $\exists\ x_0 \in \Re$ for which $\pi_F(x_0) = 1$.

III. $Supp\left(F\right) = \{x \in \Re: \pi_F(x) > 0\}$ is the support of F.

IV. $\pi_F(x)$ is an upper semi- continuous (i. e., for each $\alpha \in (0,1)$, the $\alpha -$ cut set $F_\alpha = \{x \in \Re: \pi_F \geq \alpha\}$ is closed.

Definition 4 ([14]). A fuzzy number $F = (a, b, c)$ is called a triangular fuzzy number if its membership function is given by

$$\mu_F(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \leq x \leq b, \\ \dfrac{c-x}{c-b}, & b \leq x \leq c, \\ 0, & x \geq c. \end{cases}$$

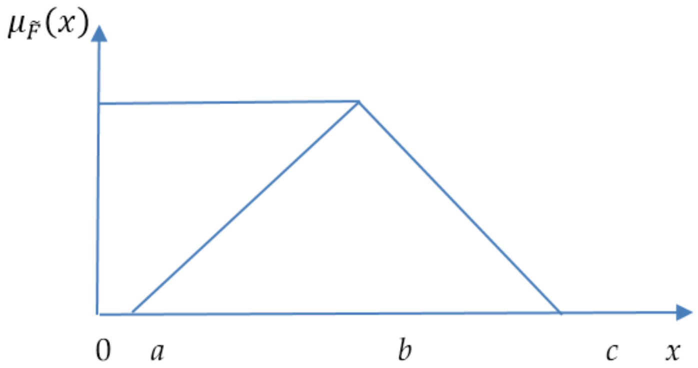

Fig. 1. Triangular fuzzy number representation.

Definition 5. Let $F = (a, b, c)$, and $G = (e, f, g)$ be two triangular fuzzy numbers. The arithmetic operations on$F$, and G are:

I. Addition: $F \bigoplus G = (a + e, b + f, c + g)$.

II. Subtraction: $F \ominus G = (a - g, b - f, c - e)$.

III. Inverse: $F^{-1} = \left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a}\right)$.

IV. Symmetric (image): $-\left(F\right) = (-c, -b, -a)$.

Definition 6. The associated ordinary number for the triangular fuzzy number $F = (a, b, c)$ is defined as $\widehat{F} = \frac{a+4b+c}{6}$.

2.1 | Notation

In the ZBB, the following notation may be used

$A(a^l, a^c, a^u)$: Triangular fuzzy number.

a^l, a^u: Bounds of presumption to least level ($\alpha = 0$).

a^c: The mode of presumption to maximal level ($\alpha = 1$).

El-Morsy| Com. Alg. Num. Dim. 1(4) (2022) 147-154

$\widetilde{H}$: Fuzzy threshold on resource availability.

3 | Numerical Example

Consider a company with four decision centers U, V, W, and Z as illustrated in *Fig. 1*. Assume that the three possible budgets, $U_0 < U_1 < U_2$, have been defined for U; two possible budgets, $V_0 < V_1$ for V;

four possible budgets, $W_0 < W_1 < W_2 < W_3$, for W; and three possible budgets, $Z_0 < Z_1 < Z_2$, for Z.

Budgets with an index 0 represent the minimal budget (i.e., under this allocation the center will be deleted), budgets with index 1 represent the normal budgets, while with indices 2, 3… are improved budgets. Here, the budgets are defined and established for a year at a time.

The ordered budgets can be represented by a flow graph. The decision-making group choose the budget beginning with the index 0. If a budget of index j is used for a center, it includes of course, the budget on the index i, $i < j$ as well.

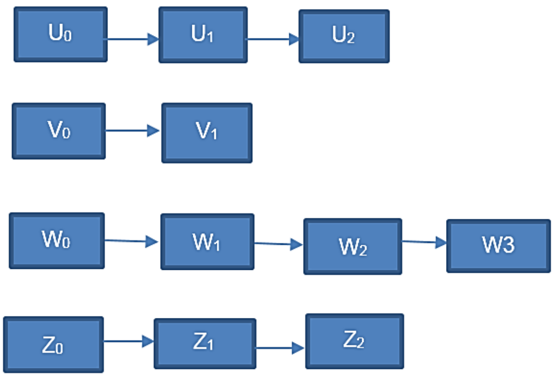

Fig. 2. Four decision centers in a company for zero base budgeting process.

$$
\begin{aligned}
U_0 &= 100, 110, 125),\\
U_1 &= 120, 125, 140),\\
U_2 &= 150, 160, 190),\\
V_0 &= 80, 100, 130),\\
V_1 &= 130, 140, 170),\\
W_0 &= 40, 50, 70),\\
W_1 &= 50, 65, 80),\\
W_2 &= 65, 80, 95),\\
W_3 &= 80, 100, 120),\\
Z_0 &= 100, 120, 130),\\
Z_1 &= 125, 140, 170),\\
Z_2 &= 150, 170, 180).
\end{aligned} \tag{1}
$$

Now, let us consider various steps in the budget selection process which yield the cumulative budget in a linear order as follows:

It is obvious that in cumulative process, the lower category resulted from any decision centers will be dropped.

$$
\begin{aligned}
&W_0 = \ 40, 50, 70),\\
&W_0 \oplus Z_0 = \ 40, 50, 70) \oplus \ 100, 120, 130) = \ 140\ 170, 200),\\
&W_1 \oplus Z_0 = \ 50, 65, 80) \oplus \ 100, 120, 130) = \ 150, 185, 210),\\
&U_0 \oplus W_1 \oplus Z_0 = \ 250, 295, 335),\\
&U_1 \oplus W_1 \oplus Z_0 = \ 270, 310, 350),\\
&U_1 \oplus V_0 \oplus W_1 \oplus Z_0 = \ 350, 410, 480),\\
&U_2 \oplus V_0 \oplus W_1 \oplus Z_0 = \ 380, 445, 530),\\
&U_2 \oplus V_0 \oplus W_2 \oplus Z_0 = \ 395, 460, 545),\\
&U_2 \oplus V_1 \oplus W_2 \oplus Z_0 = \ 445, 500, 585),\\
&U_2 \oplus V_1 \oplus W_3 \oplus Z_0 = \ 460, 520, 610),\\
&U_2 \oplus V_1 \oplus W_3 \oplus Z_1 = \ 485, 540, 650),\\
&U_2 \oplus V_1 \oplus W_3 \oplus Z_2 = \ 510, 570, 660).
\end{aligned} \tag{2}
$$

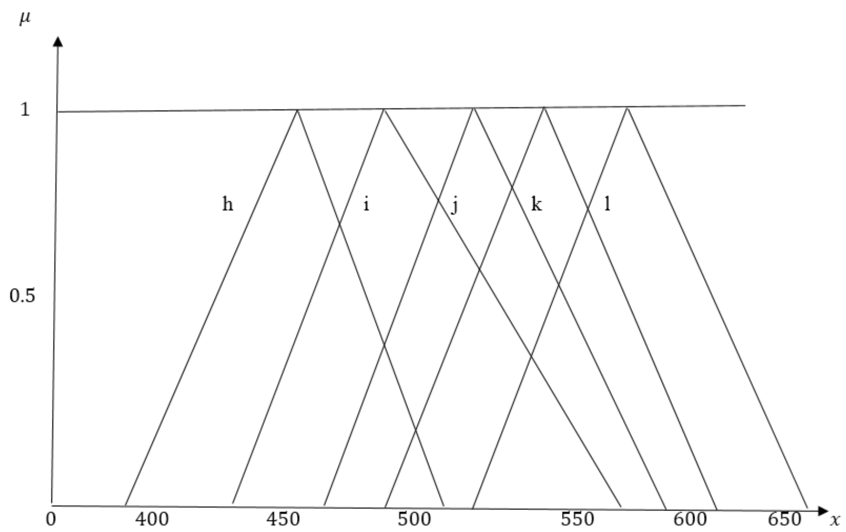

Fig. 3. Triangular fuzzy numbers representation of cumulative budgets.

Here, a threshold must be introduced because the total budget which available the company is limited. So, we take a fuzzy threshold *L* instead of its crisp:

$\forall\ x \in \Re^+$:

$$
\mu_L\ x) = \begin{cases} 1, & x \le 500,\\ \frac{600-x}{100}, & 500 \le x \le 600,\\ 0, & x \ge 600. \end{cases} \tag{3}
$$

Let us now compute the possibility of each cumulative budget taking *L* as law of possibility that is to establish the cumulative budget which can be agreed upon.

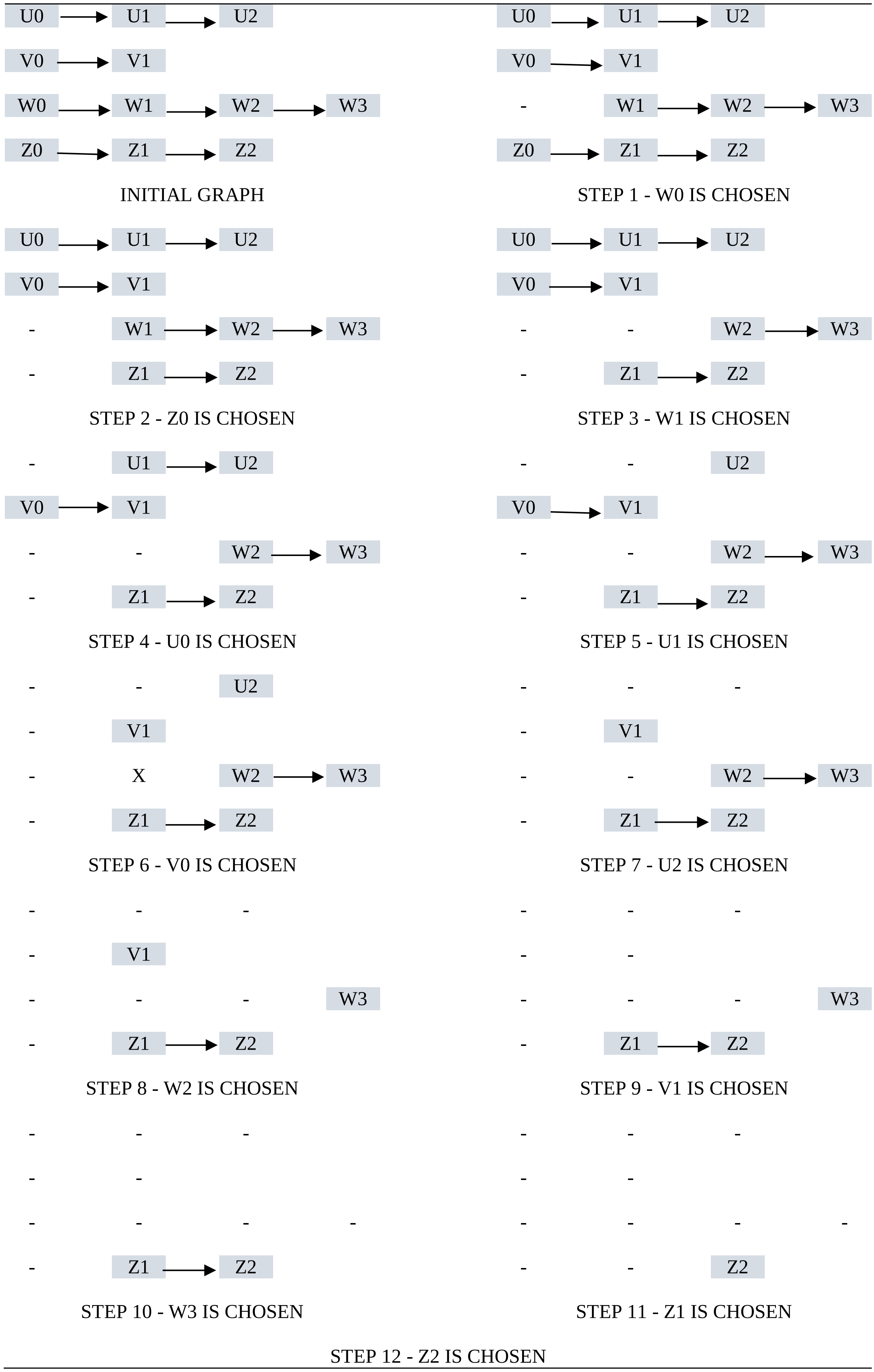

Fig. 4. Various steps in the budget selection process.

Definition 7. The possibility of X is defined as

$$\forall \ x \in \Re^+:$$

$$\text{Poss } X) = \bigvee_x \mu_X \ x) \wedge \mu_L(x)). \quad (4)$$

Based on definion7, the cumulative budget is

$Poss\ U_2 \oplus V_0 \oplus W_2 \oplus Z_0) = 1$, cumulative budget h,

$Poss\ U_2 \oplus V_1 \oplus W_2 \oplus Z_0) = 1$, cumulative budget I,

$Poss\ U_2 \oplus V_1 \oplus W_3 \oplus Z_0) = .87$, cumulative budget j,

$Poss\ U_2 \oplus V_1 \oplus W_3 \oplus Z_1) = .74$, cumulative budget k,

$Poss\ U_2 \oplus V_1 \oplus W_3 \oplus Z_2) = .53$, cumulative budget l.

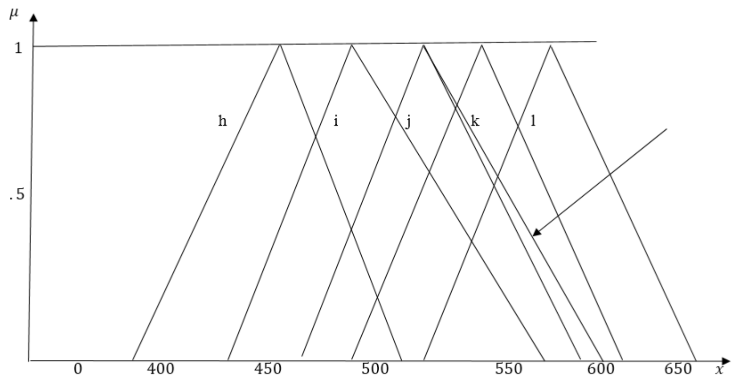

Fig. 5. Threshold budget position in relation to cumulative budgets.

4 | An Alternative Approach

For people who need to be more precise in their requirements, we propose the following alternative method.

I. Make the optimization with the associated ordinary number.
II. Consider the intervals of confidence at the leve $\alpha = 0$ .l.
III. Consider various steps in the budget selection process which yield the cumulative budget in a linear order as follows:

$$W_0 = [40, 70],$$

$$W_0 \oplus Z_0 = [40, 70] \oplus [100, 130] = [140 \ \ 200],$$

$$W_1 \oplus Z_0 = [50, 80] \oplus [100, 130] = [150, 210],$$

$$U_0 \oplus W_1 \oplus Z_0 = [250, 335],$$

$$U_1 \oplus W_1 \oplus Z_0 = [270, 350], \quad (5)$$

$$U_1 \oplus V_0 \oplus W_1 \oplus Z_0 = [350, 480],$$

$$U_2 \oplus V_0 \oplus W_1 \oplus Z_0 = [380, 530],$$

$$U_2 \oplus V_0 \oplus W_2 \oplus Z_0 = [395, 545],$$

$$U_2 \oplus V_1 \oplus W_2 \oplus Z_0 = [445,585],$$

$$U_2 \oplus V_1 \oplus W_3 \oplus Z_0 = [460, 610],$$

$$U_2 \oplus V_1 \oplus W_3 \oplus Z_1 = [485, 650],$$

$$U_2 \oplus V_1 \oplus W_3 \oplus Z_2 = [510, 660].$$

It is obvious that there is a small difference between *Calculations (2)* and *(5)*. This shows that the process employing triangular fuzzy numbers can be used without any problem.

5 | Results and Discussion

We find that, the budgets h and I can be agreed upon without any restriction with possibility equal 1; however, budget j is available but with a small risk (i.e., *Poss j*) = .87). The possibilities of k and l are 0.74 and 0.53; respectively, and therefore there is a large risk associated with these budgets.

6 | Conclusion

In this paper, novel method for modelling ZBB in fuzzy environment has described. Through this method one or several decision centers may be eliminated and thus only the departments which are most efficient will receive a healthy budget, giving an optimum use of available resources.

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